

MODELING THE TRANSVERSE MOMENTUM DEPENDENT PARTON DISTRIBUTIONS

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We review quark model calculations of the transverse momentum dependent parton distributions (TMDs). For the T-even TMDs, we discuss the physical origin of model relations which hold in a large class of quark models. For the T-odd TMDs we review results in a light-cone constituent quark model (CQM) with the final state interaction effects generated via single-gluon exchange mechanism. As phenomenological application, we show the good agreement between results in the light-cone CQM and available experimental data for the Collins asymmetry.

Keywords: transverse momentum dependent parton distributions, quark models, semi-inclusive deep inelastic scattering

1. Introduction

Parton distributions entering many hard and exclusive processes play a key role to describe the nonperturbative structure of hadrons. The most prominent examples are the ordinary parton distributions which describe the probability to find in a fast-moving hadron a parton with a certain fraction of the longitudinal momentum of the parent hadron. However, in addition to the information on the longitudinal momentum distribution, a complete three-dimensional picture of the nucleon also requires knowledge of the transverse motion of partons. A full account of the orbital motion, which is also an important issue to understand the spin structure of the

nucleon, can be given in terms of transverse-momentum dependent parton distributions (TMDs). There are eight leading-twist TMDs. Two of them, the Boer-Mulders and Sivers functions are T-odd, i.e. they change sign under naïve time reversal, which is defined as usual time reversal, but without interchange of initial and final states. The other six leading-twist TMDs are T-even. The latter will be reviewed in Sect. 2, discussing in particular the origin of model relations which were found in a large class of quark models^{1–7}. Understanding the physical origin of these relations provides useful insights for building up realistic quark models. Indeed, from phenomenological applications to observables one can verify to which extent the assumptions at the basis of the model relations are also satisfied in “nature”. The T-odd TMDs will be the subject of Sect. 3, where we will show predictions in a light-cone constituent quark model (CQM)⁸ which was successfully applied to describe many nonperturbative properties of the nucleon, like generalized parton distributions⁹, form factors¹⁰ and distribution amplitudes¹¹. The same light-cone CQM was also applied to describe single-spin asymmetries in semi-inclusive deep inelastic (SIDIS) which allow one to extract information on TMDs¹². In Sect. 4 we will show as an example the results for the Collins asymmetry.

2. T-even TMDs and quark model relations

In the light-cone gauge $A^+ = 0$, the physical meaning of the correlations encoded in the T-even TMDs becomes especially transparent when using the canonical expansion in terms of light-cone Fock operators for the quark fields. Restricting ourselves to the quark contribution, one finds

$$f_1^q(x, \mathbf{k}_\perp^2) = \langle P\Lambda | V^q(\tilde{k}) | P\Lambda \rangle, \quad (1)$$

$$\Lambda g_{1L}^q(x, \mathbf{k}_\perp^2) = \langle P\Lambda | A^q(\tilde{k}) | P\Lambda \rangle, \quad (2)$$

$$\frac{\mathbf{k}_\perp \cdot \mathbf{S}_\perp}{M} g_{1T}^q(x, \mathbf{k}_\perp^2) = \langle PS_\perp | A^q(\tilde{k}) | PS_\perp \rangle, \quad (3)$$

$$\Lambda \frac{\mathbf{k}_\perp}{M} h_{1L}^{\perp q}(x, \mathbf{k}_\perp^2) = \langle P\Lambda | \mathbf{T}_\perp^q(\tilde{k}) | P\Lambda \rangle, \quad (4)$$

$$\mathbf{S}_\perp h_{1T}^q(x, \mathbf{k}_\perp^2) + \frac{\mathbf{k}_\perp \cdot \mathbf{S}_\perp}{M} \frac{\mathbf{k}_\perp}{M} h_{1T}^{\perp q}(x, \mathbf{k}_\perp^2) = \langle PS_\perp | \mathbf{T}_\perp^q(\tilde{k}) | PS_\perp \rangle. \quad (5)$$

where Λ (\mathbf{S}_\perp) is the longitudinal (transverse) light-cone polarization of the nucleon, and P is the nucleon four-momentum. In Eqs. (1)–(5), the quark operators V^q , A^q , and \mathbf{T}_\perp^q can be written in terms of creation and annihilation operators of quarks with flavor q , light-cone helicity λ and

light-cone momentum $\tilde{k} = (xP^+, \mathbf{k}_\perp)$ as

$$V^q(\tilde{k}) = \sum_\lambda q_\lambda^\dagger(\tilde{k})q_\lambda(\tilde{k}), \quad A^q(\tilde{k}) = \sum_\lambda \text{sgn}(\lambda) q_\lambda^\dagger(\tilde{k})q_\lambda(\tilde{k}), \quad (6)$$

$$T_R^q(\tilde{k}) = \left[T_L^q(\tilde{k}) \right]^\dagger = 2q_+^\dagger(\tilde{k})q_-(\tilde{k}),$$

where for a generic two-component vector \mathbf{a}_\perp we have defined $a_{R,L} = a^1 \pm ia^2$. The quark operators V^q and A^q have a probabilistic interpretation since they are written just in terms of number operators $N_\lambda^q = q_\lambda^\dagger(\tilde{k})q_\lambda(\tilde{k})$. The operator \mathbf{T}_\perp^q has also a probabilistic interpretation but only when written in terms of transversely polarized operators

$$\mathbf{S}_\perp^q \cdot \mathbf{T}_\perp^q = \sum_{s_\perp} \text{sgn}(s_\perp) q_{s_\perp}^\dagger(\tilde{k})q_{s_\perp}(\tilde{k}). \quad (7)$$

Choosing the frame system such that $\hat{e}_z = \hat{P}$ and $\hat{e}_y = \hat{k}_\perp$, the T-even TMDs can be classified in terms of the quark and nucleon light-cone polarizations as summarized in Table 1.

Table 1. T-even TMDs according to quark (columns) and nucleon (rows) light-cone polarizations.

nucleon pol \ quark pol	U	L	T _y	T _x
	f_1	g_{1L}	$\frac{k_\perp}{M} h_{1L}^\perp$	
U		$\frac{k_\perp}{M} g_{1T}$	h_{1T}^+	
L				h_{1T}^-
T _y				
T _x				

Note: $h_{1T}^{\pm q} = h_1 \pm \frac{k_\perp^2}{2M} h_{1T}^\perp$.

In QCD all TMDs are independent functions. However, in a large class of quark models¹⁻⁷ there appear relations among different TMDs. In particular, for the T-even TMDs one finds *i*) three linear relations

$$C^q f_1^q + g_{1L}^q - 2h_1^q = 0, \quad (8)$$

$$g_{1T}^q + h_{1L}^{\perp q} = 0, \quad (9)$$

$$g_{1L}^q - \left[h_1^q + \frac{k_\perp^2}{2M^2} h_{1T}^{\perp q} \right] = 0, \quad (10)$$

with C^q in Eq. (8) a real number, and *ii*) a quadratic relation

$$2h_1 h_{1T}^{\perp q} = -(g_{1T}^q)^2. \quad (11)$$

Although such model relations hold in quark models based on different descriptions of the quark dynamics, their origin can be traced back to a few common assumptions for treating the quark and nucleon spin degrees of freedom. Indeed, Eqs. (8)-(11) connect the reciprocal polarizations of quark and nucleon, and we expect an underlying symmetry under rotation of the polarization degrees of freedom, common to all models. In order to describe the behavior of the polarizations under rotations, it is convenient to work in the basis of canonical (instant form) spin instead of light-cone helicity. This is because rotations in light-cone quantization depend non trivially on the interaction, while they become kinematical operators within instant-form quantization. A first common assumption to all the quark models satisfying the relations is that there is no mutual interaction among the quarks at the interaction vertex. In this case, light-cone helicity and canonical spin operators are just related by a Wigner rotation in the polarization space with axis orthogonal to both \hat{e}_z and \mathbf{k}_\perp directions

$$q_\lambda(\tilde{k}) = D_{\lambda s}^{(1/2)*} q_s(\tilde{k}), \quad D_{\lambda s}^{(1/2)*} = \begin{pmatrix} \cos \frac{\theta}{2} & \hat{k}_L \sin \frac{\theta}{2} \\ -\hat{k}_R \sin \frac{\theta}{2} & \cos \frac{\theta}{2} \end{pmatrix}, \quad (12)$$

where λ and s refer to the light-cone helicity and canonical spin, respectively, while the explicit expression of the angle θ in the different models can be found in Ref. [13]. Applying the rotation in Eq. (12) to the polarization operators in Eq. (6) and the nucleon states, one finds the expressions of the T-even TMDs in the basis of canonical spin given in Table 2. If we

Table 2. T-even TMDs according to quark (columns) and nucleon (rows) canonical polarizations.

nucleon pol \ quark pol	U	L	T_y	T_x
	f_1			
U				
L		$\cos \theta g_{1L} - \sin \theta \frac{k_\perp}{M} h_{1L}^\perp$	$\cos \theta g_{1L} + \sin \theta \frac{k_\perp}{M} h_{1L}^\perp$	
T_y		$\cos \theta \frac{k_\perp}{M} g_{1T} - \sin \theta h_{1T}^+$	$\sin \theta \frac{k_\perp}{M} g_{1T} + \cos \theta h_{1T}^+$	
T_x				h_{1T}^-

assume rotational symmetry around one of the coordinate axis, we find:

- for the \hat{e}_x axis: the probability to find a quark with spin parallel to the longitudinal direction in a longitudinally polarized nucleon is equal to the probability to find both quark and nucleon transversely polarized in the \hat{e}_y direction. As a result, one can derive from Table 2 the two linear relations of Eqs. (9) and (10);

- for \hat{e}_z axis: the momentum distribution is axially symmetric when the canonical spins of the active quark and nucleon are aligned in the same transverse direction. Using the expressions in Table 2, one finds the quadratic relation of Eq. (11).

The cylindrical symmetry around the \hat{e}_y leads to relations among TMDs which are not independent from the previous ones. The last linear relation in Eq. (8), relating unpolarized and polarized spin amplitudes, holds within more restrictive conditions. It requires spherical symmetry and $SU(6)$ symmetry for the spin-isospin dependence. We note that all the models discussed in the literature which satisfy the relations (9) and (10) among polarized TMDs and the quadratic relation (11) satisfy spherical symmetry in the instant-form, which is equivalent to assuming cylindrical symmetry around all the three spatial axis. Note that this condition is sufficient but not necessary for the validity of the relations.

3. T-odd TMDs

The gauge-link operator entering the quark correlation function which defines the TMDs is crucial to obtain non-zero T-odd quark distributions. In the light-cone CQM⁸, we consider the expansion of the gauge-link operator at leading order, taking into account the one-gluon exchange mechanism between the struck quark and the nucleon spectators described by (real) light-cone wave functions (LCWFs). This approach is complementary to a recent work⁸ where the rescattering effects are incorporated in augmented LCWFs containing an imaginary phase. The approximation of truncating the expansion of the gauge-link operator at leading order is common to most of the quark-model calculations in literature^{15,16}. However, studies beyond the one-gluon exchange approximation where one resums all order contributions have recently been presented in Ref. [17]. Following Ref. [8], one can represent the T-odd TMDs in terms of overlap of light-cone amplitudes corresponding to different orbital angular momentum. Both T-odd TMDs are obtained from the interference of wave components which differ by one unit of orbital angular momentum. In particular, we found that the Sivers function for both up and down quarks is dominated by the interference of S and P wave components, while the $P - D$ wave interference terms contribute at most by 20%. On the other side, the relative weight of the $P - D$ wave interference terms increases in the case of the Boer-Mulders function, in particular for the down-quark component. In Fig. 1 we show the results for the first transverse-momentum moments of the Sivers and

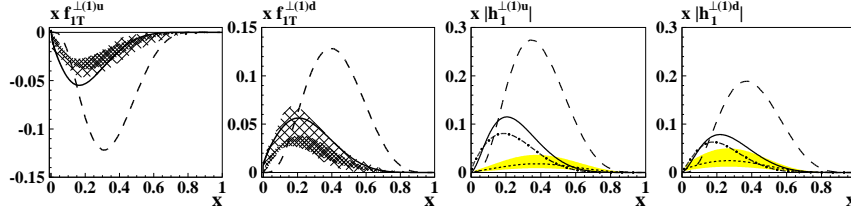


Fig. 1. Results for the first transverse-momentum moment of the Siverson and Boer-Mulders functions for up and down quarks, as function of x . See text for the explanation of the different curves.

Boer-Mulders functions. The dashed curves correspond to the results at the hadronic scale of the model. The solid curves are the results evolved to $Q^2 = 2.5 \text{ GeV}^2$. Since the exact evolution equations for the T-odd quark distributions are still under study, we used those evolution equations which seem most promising to simulate the correct evolution. We evolved the first transverse-momentum moment of the Siverson function by means of the evolution pattern of the unpolarized parton distribution, while for the first transverse-momentum moment of the Boer-Mulders we used the evolution pattern of the transversity. After evolution, the model results are consistent with the available parametrizations. For the Siverson function, the lighter and darker shaded areas are the uncertainty bands due to the statistical error of the parametrizations of Ref. [18] and Ref. [19], respectively, which refer to an average scale of $Q^2 = 2.5 \text{ GeV}^2$. For the Boer-Mulders function, the dashed-dotted curves are the results of the phenomenological parametrization of Refs. [20,21] at the average scale of $Q^2 = 2.4 \text{ GeV}^2$, and the short-dashed curves show the results of Refs. [22,23] valid at $Q^2 \approx 1 \text{ GeV}^2$, with the shaded area describing the variation ranges allowed by positivity bounds.

We also note that a non trivial constraint in model calculations of the Siverson function is given by the Burkardt sum rule²⁴, which corresponds to require that the net (summed over all partons) transverse momentum due to final-state interaction is zero. Our model calculation of the Siverson function reproduces exactly this sum rule.

4. Results for the Collins asymmetry

In Ref. [12] the present results for the T-even TMDs were applied to estimate azimuthal asymmetries in SIDIS, discussing the range of applicability of the model, especially with regard to the scale dependence of the ob-

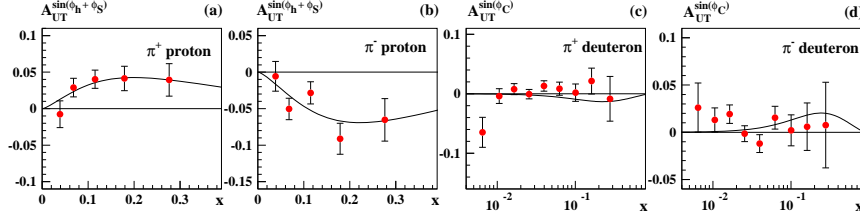


Fig. 2. The single-spin asymmetry $A_{UT}^{\sin(\phi_h + \phi_S)} \equiv -A_{UT}^{\sin \phi_C}$ in DIS production of charged pions off proton and deuterium targets, as function of x . The solid curves are obtained using the light-cone CQM predictions for $h_1(x, Q^2)$ from Refs. [3,30]. The proton target data are from HERMES²⁸, the deuterium target data are from COMPASS²⁹.

servables and the transverse-momentum dependence of the distributions. Here we review the results for the Collins asymmetry $A_{UT}^{\sin(\phi + \phi_S)}$, due to the Collins fragmentation function and to the chirally-odd TMDs h_1 . For the Collins function we use the results extracted in Ref. [25]. In the denominator of the asymmetry we take f_1 from Ref. [26] and the unpolarized fragmentation function from Ref. [27], both at the scale $Q^2 = 2.5 \text{ GeV}^2$.

In Fig. 2 the results for the Collins asymmetry in DIS production of charged pions off proton and deuterium targets are shown as function of x . The model results for h_1 evolved from the low hadronic scale of the model to $Q^2 = 2.5 \text{ GeV}^2$ ideally describe the HERMES data²⁸ for a proton target (panels (a) and (b)). This is in line with the good agreement between our model predictions³⁰ and the phenomenological extraction of the transversity and the tensor charges.³¹ Our results are also compatible with the COMPASS data²⁹ for a deuterium target (panels (c) and (d)) which extend down to much lower values of x .

Acknowledgments

B.P. acknowledges a fruitful collaboration with S. Boffi, A. Efremov, P. Schweitzer, and F. Yuan with whom some of the results presented in this paper have been obtained. This work was supported in part by the Research Infrastructure Integrating Activity “Study of Strongly Interacting Matter” (acronym HadronPhysics2, Grant Agreement n. 227431) under the Seventh Framework Programme of the European Community, by the Italian MIUR through the PRIN 2008EKLACK “Structure of the nucleon: transverse momentum, transverse spin and orbital angular momentum”.

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